CALCULATION OF HEATING AND COOLING OF PRODUCTS MOVING LONGITUDINALLY IN A FLUIDIZED BED

A. P. Baskakov and E. A. Kondrat'ev

UDC 621.785:66.096.5

An analysis is made of the problem of the heating and cooling of a metal product moving through two sequentially arranged chambers containing a fluidized bed in the presence of a return flow of heat by thermal conduction along the axis of the product.

As shown by an experiment on the industrial exploitation of conveyer muffle furnaces containing fluidized beds for the thermal treatment of pipes [1, 2], with the help of automatic regulators different temperatures (800-1100°C) are stably maintained in different zones of the furnace even without the presence of transverse partitions between them. Almost any given temperature drop can be obtained in the presence of partitions between the zones. In this connection the thermal treatment of metal products in accordance with a complicated schedule, such as heating—cooling, can be accomplished in conveyer furnaces containing fluidized beds.

If the thermal conductivity of the metal is high enough the rate of cooling of the section of a long pipe (or rod) entering the cooling zone from the heating zone can differ from the rate of cooling of a short segment of the same pipe carried from one zone to the other. In a number of cases, such as in the presence of hardening, the high thermal conductivity of the metal, especially with large pipe or rod cross sections, can lead to a marked decrease in the cooling rate in conveyer assemblies in comparison with the rate obtained on separate specimens.

Heating and cooling under the conditions described are analyzed in the present article. The following assumptions are adopted in the solution of the stated problem.

A product of infinite length (pipe, rod, billet, etc.) moves through a heating zone with a constant bed temperature t_h^b along the length l_h and a cooling zone with a constant bed temperature t_c^b along the length l_c. We assume that the process is established in the sense that the temperatures in each section of the assembly are taken as independent of time. The product is assumed to be a thin body in thermal engineering terms, which allows one to neglect the temperature variation in its cross section. In the definitions adopted in Fig. 1 the process is described by equations obtained on the assumption of constancy of all the thermal engineering constants [3].

In the heating zone

$$\frac{d^2\vartheta_h}{dx^2} - A\frac{d\vartheta_h}{dx} - B_h\vartheta_h = 0. \quad (1)$$

In the cooling zone

$$\frac{d^2\vartheta_c}{dx^2} - A\frac{d\vartheta_c}{dx} - B_c\vartheta_c = 0. \quad (2)$$

In the sections where heat exchange with the product is practically absent (before the assembly, in the intermediate wall, and after the furnace), the temperature field in the product is obtained by the equation

$$\frac{d^2t}{dx^2} - A\frac{dt}{dx} = 0. \quad (3)$$

S. M. Kirov Ural Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 5, pp. 778-786, November, 1975. Original article submitted September 16, 1974.

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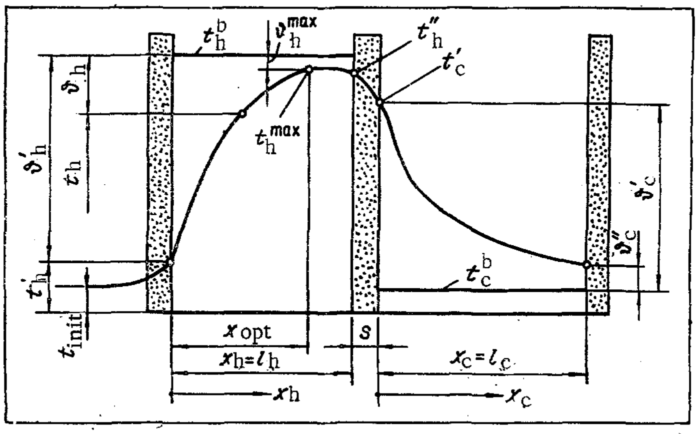

Fig. 1. Temperature variation of an infinitely long product moving through heating and cooling chambers in the presence of a return flow of heat by thermal conduction along the axis of the product.

Since in the problem under consideration the temperature and the heat flux transferred by thermal conduction along the axis of the product must be continuous functions of x, Eqs. (1), (2), and (3) are interconnected by the conditions of equality of the temperatures and of their derivatives with respect to x at the junctions of the zones, i.e., at $x_h = 0$, $x_h = l_h$, and $x_c = 0$. As additional boundary conditions we assume that

$$t_h = t_{init} \text{ and } \frac{dt}{dx} = 0, \text{ at } x_h = -\infty,$$
$$\text{at } x_c = \infty. \quad \frac{dt}{dx} = 0. \tag{4}$$

With the boundary conditions (4) it follows from (3) that $dt/dx = 0$ for all $x \geq (l_h + s + l_c)$. Physically this means that in the absence of heat transfer from the product to the surrounding medium at the exit from the assembly the heat flux along it equals zero. With $\lambda \neq 0$ this signifies the absence of a temperature gradient. In this connection one can write

$$\frac{dt}{dx} = 0 \quad \text{at } x_c = l_c \,. \tag{4'}$$

At the entrance to the assembly all the heat entering through thermal conduction counter to the moving product goes into heating it, and therefore

$$A(t'_h - t_{init}) = \left.\frac{dt}{dx}\right|_{x_h} = 0. \tag{5}$$

In order to simplify the calculations we will first take t'_h, t''_h, and t'_c as known quantities and then find them by equating the corresponding solutions to each other. We introduce the dimensionless quantities

$$\psi_h = \frac{B_h}{A^2} = \frac{\alpha_h P \lambda}{F(c\rho w)^2}; \tag{6}$$

$$\delta_h = \frac{B_h}{A} l_h = \frac{\alpha_h P l_h}{F c\rho w}; \tag{7}$$

$$\bar{X}_h = \frac{x_h}{l_h}. \tag{8}$$

Then the solution of Eq. (1) with

$$\bar{X}_h = 0 \quad \vartheta_h = \vartheta'_h; \quad \bar{X}_h = 1 \quad \vartheta_h = \vartheta''_h$$

will have the form

$$\vartheta_h = \left\{\vartheta_h''\left[1-\exp\left(-\frac{\delta_h}{\psi_h}\sqrt{1+4\psi_h}\,\bar{X}_h\right)\right]\exp\left[-\frac{\delta_h}{2\psi_h}\left(1+\sqrt{1+4\psi_h}\right)(1-\bar{X}_h)\right\}\times$$
$$\times\left\{1-\exp\left(-\frac{\delta_h}{\psi_h}\sqrt{1+4\psi_h}\right)\right\}^{-1}+\left\{\vartheta_h'\left\{1-\exp\left[-\frac{\delta_h}{\psi_h}\sqrt{1+4\psi_h}\,(1-\bar{X}_h)\right]\right\}\times\right.$$
$$\left.\times\exp\left[-\frac{\delta_h}{2\psi_h}\left(\sqrt{1+4\psi_h}-1\right)\bar{X}_h\right]\right\}\left\{1-\exp\left(-\frac{\delta_h}{\psi_h}\sqrt{1+4\psi_h}\right)\right\}^{-1}. \tag{9}$$

Calculations show that $\delta_h/\psi_h >> 1$ in the majority of cases of heating. Under these conditions the second term in the denominator of Eq. (9) is equal to zero and the equation for ϑ_h is considerably simplified.

Usually the criterion δ_h varies in the range of 1.5-5.0, and therefore $\psi_h << 1$. By expanding $\sqrt{1 + 4\psi_h}$ in a series and keeping only the first two terms one can obtain in place of (9)

$$\vartheta_h \simeq \vartheta_h' \exp(-\delta_h \bar{X}_h). \tag{10}$$

In this case at the end of the heating zone (at $\bar{X}_h = 1$) we will have

$$\vartheta_h^{max} \simeq \vartheta_h' \exp(-\delta_h). \tag{11}$$

Here it was considered that $\vartheta_h'' \ll \vartheta_h'$ and therefore the first term in Eq. (9) can be neglected in comparison with the second for all values of $\bar{X}_h$ much smaller than unity. As $\bar{X} \to 1$, however, the second term in Eq. (9) approaches zero and the exponent of the first term approaches unity, and therefore it cannot be neglected, since in this case $\vartheta_h = \vartheta_h''$. Actually, Eqs. (10)-(11) correspond to the case of heating of a product with $\lambda = 0$; they cannot be used when $X_h \simeq 1$.

By finding dt/dx from Eq. (9), substituting the value found into (5), and performing a series of transformations with allowance for the fact that $\delta_h/\psi_h >> 1$ while $\exp(-\delta_h/\psi_h)$ and $\exp[-(\delta_h/\psi_h)\sqrt{1 + 4\psi_h}]$ are small in comparison with $\exp(\delta_h/\psi_h)\sqrt{1 + 4\psi_h}$, we obtain an expression for computing t_h':

$$\vartheta_h' \simeq \vartheta_{init}\frac{1}{1+\psi_h}. \tag{12}$$

The temperature t_h'' does not enter into Eq. (12). Physically, this means that the conditions of heat exchange at the end of the heating zone have almost no effect on the temperature gradient at its entrance.

The solution of Eq. (2) with

$$\bar{X}_c = 0 \quad \vartheta_c = t_c' - t_c^b = \vartheta_c'; \; \bar{X}_c = 1 \quad \vartheta_c = \vartheta_c''$$

for the cooling zone will have the form

$$\vartheta_c = \vartheta_c' \exp\left[-\frac{\delta_c}{2\psi_c}\left(\sqrt{C}-1\right)\bar{X}_c\right]\frac{1-\dfrac{1-\sqrt{C}}{1+\sqrt{C}}\exp\left[-\dfrac{\delta_c}{\psi_c}\sqrt{C}\,(1-\bar{X}_c)\right]}{1-\dfrac{1-\sqrt{C}}{1+\sqrt{C}}\exp\left(-\dfrac{\delta_c}{\psi_c}\sqrt{C}\right)}. \tag{13}$$

Here $\sqrt{C} = \sqrt{1 + 4\psi_c}$.

Since for the cooling of metal products in conveyer assemblies in practice the cooling zone is taken as rather large, while $\delta_o/2\psi_o >> 1$, i.e., $\exp[-(\delta_c/\psi_c)\sqrt{C}] \simeq 0$, Eq. (13) can be simplified, since the denominator will equal unity, while the second term in the numerator equals zero. Then the equation takes the form

$$\vartheta_c \simeq \vartheta_c' \exp\left[-\frac{\delta_c}{2\psi_c}\left(\sqrt{C}-1\right)\right]\bar{X}_c. \tag{14}$$

If $\lambda = 0$, then we obtain the usual expression

$$\vartheta_c \simeq \vartheta_c' \exp(-\delta_c \bar{X}_c). \tag{15}$$

The temperature distribution at the end of the heating section and at the start of the cooling section is of the greatest interest for an analysis of the hardening process, since the temperature t_c' at the entrance to the cooling zone when the thermal conductivity of the product is high will theoretically always be lower than the maximum heating temperature t_h^{max} of the product, as is seen from Fig. 1. In this connection it is interesting to determine at what distance from the start of the heating zone one observes the maximum temperature t_h^{max} of the product and how sharply its temperature decreases in the initial section of the cooling chamber.

By differentiating Eq. (14) and substituting $\bar{X}_c = 0$ after the differentiation we will have

$$\left.\frac{dt_c}{dx}\right|_{x_c=0} = \left.\frac{d\vartheta_c}{dx}\right|_{x_c=0} = -\vartheta_c' \frac{\delta_c}{2\psi_c} (\sqrt{C} - 1). \tag{16}$$

Taking the derivative dt/dx at $\bar{X}_h = 1$ from Eq. (9), we obtain

$$-\left.\frac{dt_h}{dx}\right|_{\bar{X}_h=1} = \left.\frac{d\vartheta_h}{dx}\right|_{\bar{X}_h=1} = \vartheta_h'' \left[\frac{\delta_h}{2\psi_h} (\sqrt{H} + 1) + (\sqrt{H} - 1) \exp\left(-\frac{\delta_h}{\psi_h} \sqrt{H}\right] - \vartheta_h' \frac{\delta_h}{\psi_h} \sqrt{H} \quad \exp\left[-\frac{\delta_h}{2\psi_h} (\sqrt{H} - 1)\right]. \tag{17}$$

For simplicity we will take the thickness s of the partition between chambers as equal to zero. Equating the derivatives from (16) and (17) under the condition that $t_h'' = t_c'$ and neglecting $\exp(-\delta_h/\psi_h\sqrt{H})$, we obtain

$$t_h'' = t_c' = \left\{ t_h^b \frac{\delta_h}{2\psi_h} (\sqrt{H} + 1) + t_c^b \frac{\delta_c}{2\psi_c} (\sqrt{C} - 1) - \vartheta_h' \frac{\delta_h}{\psi_h} \times \sqrt{H} \exp\left[-\frac{\delta_h}{2\psi_h} (\sqrt{H} - 1)\right] \right\} \left\{ \frac{\delta_h}{2\psi_h} (\sqrt{H} + 1) + \frac{\delta_c}{2\psi_c} (\sqrt{C} - 1) \right\}^{-1}. \tag{18}$$

Equation (18) is simplified if $\sqrt{H} = \sqrt{1 + 4\psi_h}$ and $\sqrt{C} = \sqrt{1 + 4\psi_c}$ are expanded in series, keeping only the first two terms:

$$t_h'' = t_c' = \frac{t_h^b \left(\frac{1}{\psi_h} + 1\right) + t_c^b \frac{\delta_c}{\delta_h} - \vartheta_h' \left(\frac{1}{\psi_h} + 2\right) \exp(-\delta_h)}{\left(\frac{1}{\psi_h} + 1\right) + \frac{\delta_c}{\delta_h}}. \tag{19}$$

In furnaces in which the temperature of the medium does not greatly exceed the maximum temperature of the product (which is characteristic for furnaces containing fluidized beds), and even more when a technological holding time is necessary, $\delta_h \gg 1$. Under these conditions the last term in Eq. (19) approaches zero and then the temperature of the product at the boundary of the zones proves to be a function only of ψ_h and $\delta_c/\delta_h = \alpha_c/\alpha_h$. For larger values of δ_h the conditions at the entrance to the heating chamber no longer affect t_h''.

By equating the derivative dt_h/dx found from (9) to zero, after transformations we find the distance $\bar{X}_{opt}$ from the entrance to the heating chamber which corresponds to the maximum temperature of the product:

$$\bar{X}_{opt} = 1 - \frac{1}{\left(\frac{1}{\psi_h} + 1\right)\delta_h} \ln\left(\frac{1}{\psi_h} + 1\right)\left(\frac{\vartheta_h''}{\vartheta_h'} \exp \delta_h - 1\right). \tag{20}$$

Calculations show that in all cases (if holding at the heating temperature is not required) it is inadvisable to have δ_h larger than 3-5, since at these values the maximum temperature of heating of the product is already almost equal to the temperature of the heating medium. It is seen from Fig. 2 that the presence of a heating zone decreases the temperature gradient in the product during cooling more markedly, the larger α_0, although with the values of the parameters ψ_h, δ_h, ψ_c, and δ_c which are usual in practice this effect is small and only noticeable in the immediate vicinity of the dividing wall.

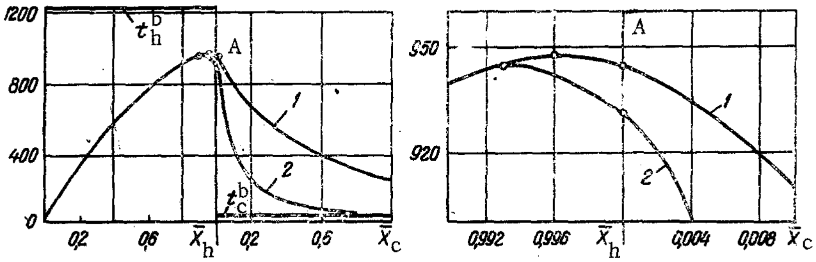

Fig. 2. Effect of parameters δ_h and δ_c on the cooling rate at the start of the cooling zone and the optimum length of the heating zone in this case: 1) $\delta_h = \delta_c = 1.5$, $\delta_h/\psi_h = \delta_c/\psi_c = 200$, $t_h' = t_{init} = 20°C$, $t_h^b = 1220°C$, $t_c^b = 20°C$; 2) $\delta_h = 1.5$ and $\delta_c = 10$, $\delta_h/\psi_h = 200$, $\delta_c/\psi_c = 30$, $t_h' = t_{init} = 20°C$, $t_h^b = 1220°C$, $t_c^b = 20°C$.

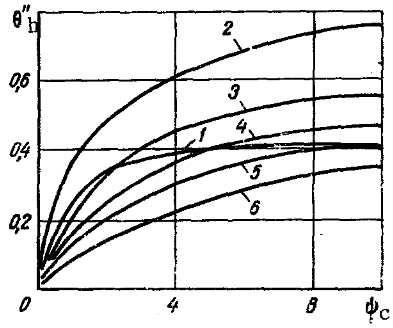

Fig. 3. Dependence of θ_h'' on the parameter ψ_c: 1) $\psi_c = \psi_h$; 2) $\psi_h = 0$; 3) $\psi_h = 2$; 4) $\psi_h = 5$; 5) $\psi_h = 10$; 6) $\psi_h = 20$. In all cases s is equal to zero. $\theta_h'' = (t_h^b - t_h'')/(t_h^b - t_c^b)$.

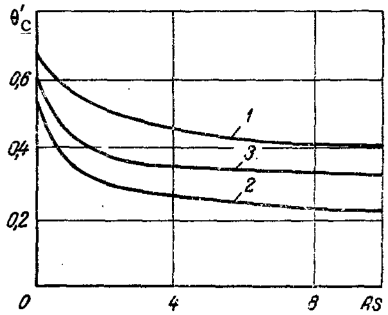

Fig. 4. Dependence of θ_c' on the dimensionless thickness As of the dividing wall for different parameters ψ_h and ψ_c: 1) $\psi_h = \psi_c = 2$; 2) $\psi_h = \psi_c = 10$; 3) $\psi_h = 2, \psi_c = 5$. $\theta_c' = (t_h^b - t_c')/(t_h^b - t_c^b)$.

Let us analyze the effect of the thickness s of the dividing partition. Since in the majority of cases of practical interest the conditions at the entrance to the heating chamber and at the exit from the cooling chamber do not alter the effect connected with the thermal conduction of the product, we can use the solutions of Eqs. (1) and (2) for heating and cooling chambers of infinite lengths l_h and l_c with the usual boundary conditions ($t_h = t_h''$ at $x_h = 0$; $t_c = t_c'$ at $x_c = 0$; the temperatures are finite at $x_h = -\infty$ and $x_c = \infty$):

$$\vartheta_h = \vartheta_h'' \exp\left[-\frac{1}{2} A x_h \left(1 + \sqrt{H}\right)\right], \tag{21}$$

$$\vartheta_c = \vartheta_c' \exp\left[-\frac{1}{2} A x_c \left(\sqrt{C} - 1\right)\right]. \tag{22}$$

Here x_h is reckoned from the left wall of the partition and x_c from the right wall. The temperature distribution in the product in the section of the dividing wall of thickness s (Fig. 1) is described by Eq. (23), obtained from (3) with the former boundary conditions ($t_s = t_h''$ at $x_s = 0$ and $t_s = t_c'$ at $x_s = s$):

$$t_s = \frac{(t_c' - t_h'') \exp A x_s + t_h'' \exp As - t_c'}{\exp As - 1}. \tag{23}$$

As the additional boundary conditions needed for the determination of t_h'' and t_c' we use, as earlier, the equality of derivatives

$$\left.\frac{dt_h}{dx}\right|_{x_h=0} = \left.\frac{dt_s}{dx}\right|_{x_s=0}; \quad \left.\frac{dt_s}{dx}\right|_{x_s=s} = \left.\frac{dt_0}{dx}\right|_{x_h=0} \tag{24}$$

Differentiating (21), (22), and (23), with respect to x_h, x_c, and x_s, respectively, equating the derivatives in accordance with (24), and solving the system of equations obtained relative to t_h'' and t_c', we find

$$\theta_h''=\frac{\vartheta_h''}{\vartheta_b}=\frac{t_h-t_h''}{t_h^b-t_c^b}=\frac{1}{\frac{\sqrt{H}+1}{\sqrt{C}-1}\exp As+\frac{1}{2}(\sqrt{H}+1)(\exp As-1)+1}, \tag{25}$$

$$\theta_c'=\frac{\vartheta_c'}{\vartheta_b}=\frac{t_h^b-t_c'}{t_h^b-t_c^b}=\frac{1}{1+\frac{1}{2}(\sqrt{C}-1)[1-\exp(-As)]+\frac{\sqrt{C}-1}{\sqrt{H}+1}\exp(-As)}. \tag{26}$$

If the thickness of the dividing wall between zones $s \to 0$, we will have

$$\frac{\vartheta_h''}{\vartheta_b}=\frac{t_h^b-t_h''}{t_h^b-t_c^b}=\frac{\sqrt{C}-1}{\sqrt{H}+\sqrt{C}}=\frac{\sqrt{1+4\psi_c}-1}{\sqrt{1+4\psi_h}+\sqrt{1+4\psi_c}}. \tag{27}$$

From Fig. 3, plotted for the case of $s = 0$, it is seen that with an increase in the complex ψ_c, in the heat-transfer coefficient in the cooling chamber in particular, and with the value of the parameter ψ_h constant, the temperature t_h'' of the product at the boundary of separation of the chambers decreases monotonically, the stronger,the smaller the value of ψ_h. It is interesting to note that it also decreases with a simultaneous increase in the heat-exchange intensity in both chambers, when $\psi_h = \psi_c$. This result becomes more obvious (for an infinite heating chamber in which the temperature of the product far in front of the partition reaches the value t_h^b) if one considers that an increase in ψ corresponds not only to an increase in α, but also to a decrease in the complex $\rho c_p w$. For the values of ψ_h and ψ_c most often encountered in practice the difference between t_h'' and t_h^b when $s = 0$ is small.

It is completely obvious that the difference between t_h'' and t_h^b will be even less with an increase in the thickness of the dividing wall. The difference between t_h'' and t_c' increases in this case, however. It is seen from Fig. 4 that with an increase in the thickness of the wall (the dimensionless parameter As) the temperature of the product to the right of its boundary decreases the more strongly, the larger ψ_c. The temperature distribution in the wall can be constructed from Eq. (23).

By comparing the rate of cooling of the product calculated in this way with the thermokinetic curves of the metal from which it is made it is not hard to estimate whether this rate is sufficient for the hardening of the product and to choose the conditions under which the chilling of the metal upon the transition from the heating chamber to the cooling chamber will not adversely affect the properties of the hardened product.

NOTATION

$A = c\rho w/\lambda$; $B_h = \alpha_h P/\lambda F$; $B_c = \alpha_c P/\lambda F$; c, specific heat capacity; P, perimeter of cross section of product; x_h, x_c, x_s, current coordinates in heating and cooling chambers and in dividing wall; w, velocity of movement of product; t_{init}, t_h', t_h, t_h^{max}, t_h'', t_s, t_c', t_c, temperatures of product initially, at entrance to heating chamber, currently in heating chamber, of maximum heating, at end of heating zone, in dividing wall, at start of cooling zone, and currently in cooling zone; t_h^b, t_c^b, temperatures of beds; ϑ_h', ϑ_h, ϑ_h'', ϑ_c', ϑ_c, excess temperatures; $\vartheta_{wa} = t_h'' - t_c'$, temperature drop in product in section of dividing wall; $\vartheta_b = t_h^b - t_c^b$, difference between temperatures of fluidized beds; α_h, α_c, coefficients of heat exchange between product and bed in the heating and cooling chambers, respectively; δ, $\bar{X}$, and $\psi = B/A^2$, dimensionless parameters; θ_h'', dimensionless temperature at end of heating chamber and at division of zones; θ_c', dimensionless temperature at start of cooling zone;λ, coefficient of thermal conductivity of product; ρ, density of product.

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